QUANT

In the final lap

to the CAT

By

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I wish we could come out with a '21 Question Set' or a 'LMR (Last Minute Revision) Digest' for quant section of the CAT. The reason why the above books sell like hot cakes in the market for X and XII preparation is because the syllabus is well defined and the paper pattern is standard or predictable. Both these arguments are not true for the CAT. Why go as far as class X and XII? We are very much used to studying at the last moment for our graduation exams as well. The reason is much different though. We believe, if we study well ahead of the exam, we might forget everything during the exam. How else will we be able to fill those reams of supplement to earn marks? This argument is again not true about the CAT. The big question then is, what should we do for quant in the last few days before the actual CAT?

The above question is difficult to answer because, suggesting any strategy at this moment will only serve academic purpose and not really make a major difference to your quant score. Secondly, the strategy has to be different for different students. However, the least I can do for you is help you increase the confidence with which you approach the quant section. CAT being an aptitude test, the frame of mind with which you approach the paper also makes a huge difference in your performance. I will divide this article into two parts: **1.** Understanding the CAT examiner (with respect to the quant section) and **2.** Plan of action for the last few days.

1. Understanding the CAT Examiner

We have heard arguments like the "CAT is unpredictable", "Expect the unexpected", "Be prepared for Surprises". All of these arguments are true, but only about the structure of the CAT and not the form! Let me explain. Number of questions, Number of Answer options, Differential v/s Uniform marking, Section wise breakup – all are unpredictable. In fact, this is the first year when the IIM authorities have claimed that the CAT will have 60 to 70 questions. However, if you were to analyse the last few years CAT papers, especially the quant section, there is a pattern that emerges. For that let us first have a look at the breakup of the quant section in the last few years:

Topics	2004	2005	2006	2007	2008	Total
Number System	3	7	7	4	4	**25**
Algebra	9	5	7	13	5	**39**
Geometry	10	10	4	3	5	**32**
Permutation Combination	4	4	2	0	4	**14**
Others	9	4	5	5	7	**30**
Total	**35**	**30**	**25**	**25**	**25**	**140**

Note:

1. 'Others' includes topics like Ratio-Proportion, Percentages, Profit-Loss-Discount, Speed-Time-Distance, Sequence Series and certain other reasoning based questions.

2. There are certain questions that require knowledge of more than one topic together. Such questions have to be classified under the topic that plays more dominant role in solving it. For example, consider the following question from CAT 2005:

A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, A and B, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track. A jogs along the rectangular track, while B jogs along the two circular tracks in a figure of eight. Approximately, how much faster than A does B have to run, so that they take the same time to return to their starting points?
(1) 3.88% *(2) 4.22%* *(3) 4.44%* *(4) 4.70%*

The above question requires you to have knowledge about Speed, Geometry and Percentages. However it has been classified under 'Geometry' as that is the more dominant logic at play. Similarly, there is an overlap between Number System and Algebra, Number System and Permutation Combination or Geometry and Speed questions.

What trend emerges from the table?

1. There is a clear bias towards Algebra. This includes topics like Equations, Inequalities, Functions, Maxima Minima and Graphs.

2. Geometry and Number System are next in line. In fact the three topics put together constitute almost 70% of the quant questions that have appeared in the last 5 years.

3. There is a sudden increase in the proportion of Permutation Combination questions in 2008. This trend is expected to continue even this year. There are two reasons for this, first it is an online friendly question (check out GRE and GMAT syllabus) and second, a variety of questions (and of different difficulty level) can be created on this topic by merely changing some information in the question, something that the CAT examiners need to this year to create 20 different versions of the paper.

2. Plan of action for the last few days

As mentioned earlier, the plan of action may change, depending on how well or how poorly are you faring in the quant. However, there is no denying the fact that there are some topics that you need to thoroughly revise and there are some typical question types that you need to practice well.

I. NUMBER SYSTEMS

A. Divisibility of Numbers:

1. *Let S be a set of positive integers such that every element n of S satisfies the conditions (a) $1000 \leq n \leq 1200$, and (b) every digit in n is odd. Then how many elements of S are divisible by 3?*

(CAT 2005)

(1) 9 *(2) 10* *(3) 11* *(4) 12*

2. *The digits of a three-digit number A are written in the reverse order to form another three-digit number B. If B > A and B – A is perfectly divisible by 7, then which of the following is necessarily true?*

(CAT 2005)

(1) 100 < A < 299 *(2) 106 < A < 305* *(3) 112 < A < 311* *(4) 118 < A < 317*

3. *Consider four digit numbers for which the first two digits are equal and the last two digits are also equal. How many such numbers are perfect squares?*

(CAT 2007)

(1) 3 *(2) 2* *(3) 4* *(4) 0* *(5) 1*

4. *Suppose, the seed of any number is defined as follows: seed(n) = n, if n < 10, and = seed(s(n)), otherwise, where s(n) indicates the sum of the digits of n. How many positive integers 'n', such that n < 500, will have seed(n) = 9?*

(CAT 2008)

(1) 39 *(2) 72* *(3) 81* *(4) 108* *(5) 55*

5. *The number of common terms in the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ... , 466 is*

(CAT 2008)

(1) 78 *(2) 19* *(3) 20* *(4) 77* *(5) 22*

B. Finding the last few digits of a mathematical expression:

1. *The right-most non-zero digit of the number 30^{2720} is _____.*

(CAT 2005)

(1) 1 *(2) 3* *(3) 7* *(4) 9*

2. *What are the last two digits of 7^{2008}?*

(CAT 2008)

(1) 21 *(2) 61* *(3) 01* *(4) 41* *(5) 81*

C. Questions based on Remainders

1. *When 2^{256} is divided by 17 the remainder would be _____.*

(CAT 2002)

(1) 1 (2) 16 (3) 14 (4) None of these

2. *What is the remainder when 4^{96} is divided by 6?* ***(CAT 2003 (Feb))***

(1) 0 (2) 2 (3) 3 (4) 4

3. *If x = $(16^3 + 17^3 + 18^3 + 19^3)$, then x divided by 70 leaves a remainder of _____.*

(CAT 2005)

(1) 0 (2) 1 (3) 69 (4) 35

4. *Let n! = 1 × 2 × 3 × ….. × n for integer n ≥ 1. If p = 1! + (2 × 2!) + (3 × 3!) + … + (10 × 10!), then p + 2 when divided by 11! leaves a remainder of _____.*

(CAT 2005)

(1) 10 (2) 0 (3) 7 (4) 1

II. ALGEBRA

A. Equations:

1. *What values of x satisfy* $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 \le 0$ *?*

(CAT 2006)

(1) $-8 \le x \le 1$ (2) $-1 \le x \le 8$ (3) $1 < x < 8$ (4) $1 \le x \le 8$ (5) $-8 \le x \le 8$

2. *The number of solutions of the equation 2x + y = 40 where both x and y are positive integers and x ≤ y is:*

(CAT 2006)

(1) 7 (2) 13 (3) 14 (4) 18 (5) 20

3. *How many pairs of positive integers m, n satisfy* $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}$ where *n* is an odd integer less than 60?

(CAT 2007)

(1) 6 (2) 4 (3) 7 (4) 5 (5) 3

4. *If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b?*

(CAT 2008)

(1) $-\frac{1}{\sqrt{3}}$ (2) –1 (3) 0 (4) 1 (5) $-\frac{1}{\sqrt{3}}$

B. Functions:

1. *If $f(x) = x^3 - 4x + p$, and $f(0)$ and $f(1)$ are of opposite signs, then which of the following is necessarily true?*

(CAT 2004)

(1) $-1 < p < 2$ (2) $0 < p < 3$ (3) $-2 < p < 1$ (4) $-3 < p < 0$

Directions for Questions 2 and 3: *Answer the questions on the basis of the information given below.*

(CAT 2004)

$f_1(x)$	$= x$	$0 \le x \le 1$
	$= 1$	$x \ge 1$
	$= 0$	otherwise

$f_2(x) = f_1(-x)$ for all x
$f_3(x) = -f_2(x)$ for all x
$f_4(x) = f_3(-x)$ for all x

2. *How many of the following products are necessarily zero for every x: $f_1(x)f_2(x)$, $f_2(x)f_3(x)$, $f_2(x)f_4(x)$?*
(1) 0 (2) 1 (3) 2 (4) 3

3. *Which of the following is necessarily true?*
(1) $f_4(x) = f_1(x)$ for all x
(2) $f_1(x) = -f_3(-x)$ for all x
(3) $f_2(-x) = f_4(x)$ for all x
(4) $f_1(x) + f_3(x) = 0$ for all x

4. *Let $g(x)$ be a function such that $g(x + 1) + g(x - 1) = g(x)$ for every real x. Then for what value of P is the relation $g(x + P) = g(x)$ necessarily true for every real x?*

(CAT 2005)

(1) 5 (2) 3 (3) 2 (4) 6

5. *A function 1 satisfies $f(1)= 3600$, and $f(1)+f(2) + ... +f(n) = n^2 f(n)$, for all positive integers $n > 1$. What is the value of $f(9)$?*

(CAT 2007)

(1) 80 (2) 240 (3) 200 (4) 100 (5) 120

6. *Let $f(x)$ be a function satisfying $f(x)f(y) = f(xy)$ for all real x, y. If $f(2) = 4$, then what is the value of $f\left(\frac{1}{2}\right)$?*

(CAT 2008)

(1) 0 (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) 1 (5) Cannot be determined

C. Maxima Minima:

1. *Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at x = 0, f(x) is______.*

(CAT 2004)

(1) maximised whenever a > 0, b > 0 *(2) maximised whenever a > 0, b < 0*
(3) minimised whenever a > 0, b > 0 *(4) minimised whenever a > 0, b < 0*

2. *Let f(x) = max(2x + 1; 3 – 4x), where x is any real number.*
Then the minimum possible value of f(x) is:

(CAT 2006)

(1) $\frac{1}{3}$ *(2) $\frac{1}{2}$* *(3) $\frac{2}{3}$* *(4) $\frac{4}{3}$* *(5) $\frac{5}{3}$*

Directions for Questions 3 and 4:

Mr. David manufactures and sells a single product at a fixed price in a niche market. The selling price of each unit is Rs. 30. On the other hand, the cost, in rupees, of producing x units is $240 + bx + cx^2$, where b and c are some constants. Mr. David noticed that doubling the daily production from 20 to 40 units increases the daily production cost by $66\frac{2}{3}\%$. However, an increase in daily production from 40 to 60 units results in an increase of only 50% in the daily production cost. Assume that demand is unlimited and that Mr. David can sell as much as he can produce. His objective is to maximize the profit.

(CAT 2007)

3. *How many units should Mr. David produce daily?*
(1) 130 *(2) 100* *(3) 70* *(4) 150* *(5) Cannot be determined*

4. *What is the maximum daily profit, in rupees, that Mr. David can realize from his business?*
(1) 620 *(2) 920* *(3) 840* *(4) 760* *(5) Cannot be determine*

5. *A quadratic function f(x) attains a maximum of 3 at x = 1. The value of the function at x = 0 is 1. What is the value of f(x) at x = 10?*

(CAT 2007)

(1) –119 *(2) –159* *(3) –110* *(4) –180* *(5) –105*

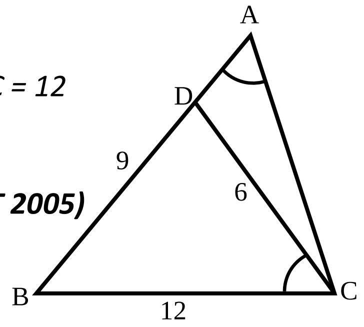
III. GEOMETRY

A. Triangles

1. *A father and his son are waiting at a bus stop in the evening. There is a lamp-post behind them. The lamp-post, the father and his son stand on the same straight line. The father observes that the shadows of his head and his son's head are incident at the same point on the ground. If the heights of the lamp-post, the father and his son are 6 metres, 1.8 metres* *and 0.9 metres respectively, and the father is standing 2.1 metres away from the post, then how far (in metres) is the son standing from his father?* ***(CAT 2004)***

(1) 0.9 *(2) 0.75* *(3) 0.6* *(4) 0.45*

2. *Consider the triangle ABC shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and ∠BCD =∠BAC What is the ratio of the perimeter of the triangle ADC to that of the triangle BDC?* ***(CAT 2005)***



(1) $\frac{7}{9}$ *(2)* $\frac{8}{9}$ *(3)* $\frac{6}{9}$ *(4)* $\frac{5}{9}$

3. *An equilateral triangle BPC is drawn inside a square ABCD. What is the value of the angle APD in egrees?*

(CAT 2006)

(1) 75 *(2) 90* *(3) 120* *(4) 135* *(5) 150*

4. *Consider obtuse-angled triangles with sides 8 cm, 15 cm and 'x' cm. If 'x' is an integer, then how many such triangles exist?*

(CAT 2008)

(1) 5 *(2) 21* *(3) 10* *(4) 15* *(5) 14*

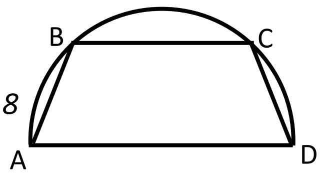
5. *In a triangle ABC, the lengths of the sides AB and AC equals 17.5 cm and 9 cm respectively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If AD = 3 cm, then what is the radius (in cm) of the circle circumscribing the triangle ABC?*

(CAT 2008)

(1) 17.05 *(2) 27.85* *(3) 22.45* *(4) 32.25* *(5) 26.25*

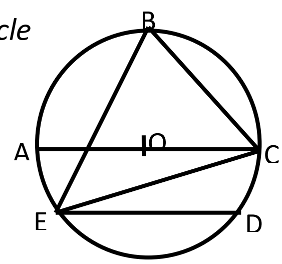
B. <u>Circles</u>

1. *On a semi-circle with diameter AD, chord BC is parallel to the diameter Further, each of the chords AB and CD are of length 2, while AD has length 8 What is the length of BC?* ***(CAT 2004)***



(1) 7.5 *(2) 7* *(3) 7.75* *(4) None of these*

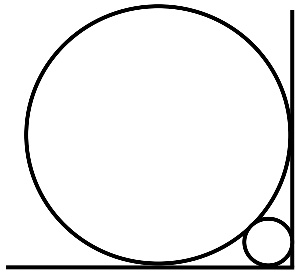
2. *In the adjoining figure, chord ED is parallel to the diameter AC of the circle If* $\angle CBE = 65^{\circ}$*, then what is the value of* $\angle DEC$*?* ***(CAT 2004)***



(1) 35° *(2)* 55°

(3) 45° *(4)* 25°

3. *A circle with radius 2 is placed against a right angle Another smaller circle is also placed as shown in the adjoining figure What is the radius of the smaller circle?* ***(CAT 2004)***



(1) $3-2\sqrt{2}$ *(2)* $4-2\sqrt{2}$
(3) $7-4\sqrt{2}$ *(4)* $6-4\sqrt{2}$

4. *Two identical circles intersect so that their centres, and the points at which they intersect, form a square of side 1 cm. The area in sq cm of the portion that is common to the circles is_____.* ***(CAT 2005)***

(1) $\frac{\pi}{4}$ *(2)* $\frac{\pi}{2}-1$ *(3)* $\frac{\pi}{5}$ *(4)* $\sqrt{2}-1$

5. *What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm?*

(CAT 2005)

(1) 1 or 7 *(2) 2 or 14* *(3) 3 or 21* *(4) 4 or 28*

6. *A semi-circle is drawn as its diameter From C, a point on AB, a line perpendicular to AB is drawn meeting the circumference of the semi-circle at D, Given that AC = 2 cm and CD = 6 cm, the area of the semi-circle (in sq cm) will be:*

(CAT 2006)

(1) 32π *(2)* 50π *(3)* 40.5π *(4)* 81π *(5) undeterminable*

7. *Two circles with centres P and Q cut each other at two distinct points A and B The circles have the same radii and neither P nor Q falls within the intersection of the circles What is the smallest range that includes all possible values of the angle AQP in degrees?*

(CAT 2007)

(1) Between 0 and 90 *(2) Between 0 and 30*
(3) Between 0 and 60 *(4) Between 0 and 75*
(5) Between 0 and 45

IV. PERMUTATION COMBINATION

1. *A new flag is to be designed with six vertical stripes using some or all of the colours – yellow, green, blue and red. The number of ways in which this can be done such no two adjacent stripes have the same colours is_____.*

(CAT 2004)

(1) 12 × 81 *(2) 16 × 192* *(3) 20 × 125* *(4) 24 × 216*

2. *There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?*

(CAT 2006)

(1) 144 (2) 180 (3) 192 (4) 360 (5) 716

3. *How many integers, greater than 999 but not greater than 40000, can be formed with the digits, 0, 1, 2, 3 and 4, if repetition of digit is allowed?*

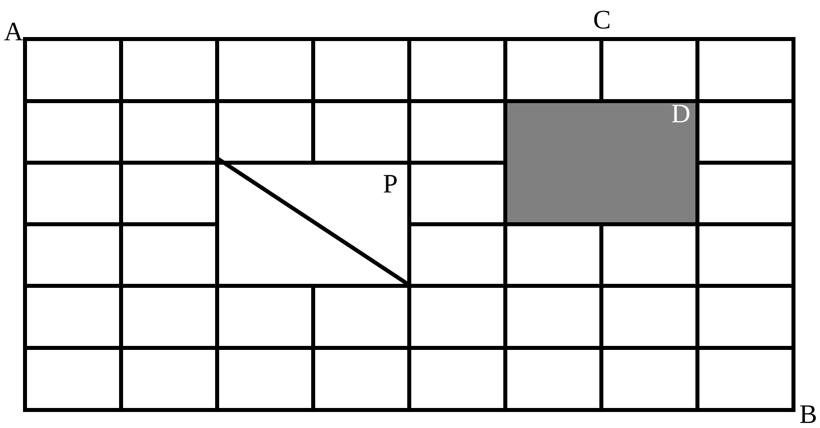
(CAT 2008)

(1) 499 (2)500 (3) 375 (4) 376 (5) 501

Direction for questions 4 and 5*: Answer the questions based on information given below.*

(CAT 2008)

The figure below shows the plan of town. The streets are right angles to each other. A rectangular park (P) is situated inside the town with a diagonal road running through it. There is also prohibited region (D) in the town.



4. *Neelam rides her bicycle from her house at A to her office at B, taking the shortest path. Then the number of possible shortest paths that she can choose is*

(1) 60 (2) 75 (3) 45 (4) 90 (5) 72

5. *Neelam rides her bicycle from her house at A to her club at C, via B taking the shortest path .then the number of possible shortest paths that can choose is*

(1) 1170 (2) 630 (3) 792 (4) 1200 (5) 936

The examples mentioned above are only the representative types from the important topics. You need to pull out more examples on the same lines from the study material you are practicing from and solve them in the last few days. As I mentioned earlier, it is tough to expect miracles at the last minute. However, even if you approach the paper with a mental satisfaction that you have revised all the important topics of quant, it could do all the difference to your confidence and the frame of mind with which you approach the quant section on the D-day.

All the best!